

Powering Through: Network Analysis of a Synthetic US Transmission Grid

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1 Abstract

The US power transmission network is critical infrastructure whose structural topology governs both normal operation and resilience under failure. This study applies network science to a synthetic 2,000-bus US transmission case from ARPA-E’s Grid Optimization Competition to test three structural hypotheses: whether the degree distribution follows a power law, whether the network exhibits small-world topology, and how it responds to random failure versus targeted attack. The network is fully connected: all 2,000 buses form a single component linked by 2,806

transmission lines, with a mean degree of 2.81. The degree distribution is heterogeneous but only weakly consistent with a power law, consistent with the physical constraints of engineered networks. The network is unambiguously small-world: clustering far exceeds the random-graph expectation while average path length remains relatively short (see [Table 2](#)). Betweenness centrality is concentrated in a small fraction of high-degree buses that bridge regional clusters. Under targeted removal of high-degree nodes, the largest connected component collapses rapidly; under random failure, the network tolerates substantially more loss before fragmenting. These findings align with the theoretical predictions for heterogeneous networks and point to a small number of structurally critical substations whose failure carries outsized systemic risk.

2 Introduction

Every time you flip a switch, electrons respond through a grid spanning hundreds of thousands of miles of high-voltage wire, balancing supply and demand across three major interconnects in real time. The US power grid is among the most complex engineered systems ever built — and yet, at the level of topology, it is a graph: nodes (substations and generating stations) connected by edges (transmission lines).

Network science offers a complementary lens to classical power systems analysis. Where load-flow and stability analysis ask how electricity moves through a given configuration, network science asks what the structure itself implies: which nodes concentrate influence, how efficiently signals propagate, and what failure modes the topology enables.

These questions gained operational urgency after two foundational results from the late 1990s. Watts and Strogatz (1998) showed that many real-world networks — from Hollywood actor collaborations to the Western US power grid — combine high local clustering with short global path lengths, a property they named “small-world.” Barabási and Albert (1999) showed that networks grown by preferential attachment develop power-law degree distributions with a few dominant hubs, calling such networks “scale-free.” Albert, Jeong, and Barabási (2000) then linked structure to resilience: scale-free networks are simultaneously robust to random node failure (random removal rarely targets a hub) and fragile to targeted attack (deliberate removal of hubs destroys connectivity quickly).

Watts and Strogatz explicitly analyzed the Western US power grid as a small-world network in their 1998 paper. Subsequent work has debated whether it is also scale-free — several studies found power-law degree distributions, while others found exponential or truncated distributions more consistent with engineering constraints Albert et al. (2004).

This study tests both properties on a synthetic 2,000-bus US transmission case from the ARPA-E Grid Optimization Competition (Birchfield et al. 2017; Coffrin et al. 2019). The real WECC topology is classified under Critical Energy Infrastructure Information (CEII) restrictions and is not publicly available. Synthetic cases generated to statistically match real US grid topology provide a principled substitute for structural analysis. Three questions guide the work:

1. Is the degree distribution consistent with a power law?
2. Does the network exhibit small-world topology?
3. How does the connected component structure respond to targeted versus random node removal?

3 Data

The analysis uses the `pglib_opf_case2000_goc` case (Coffrin et al. 2019), a synthetic 2,000-bus US transmission network developed for ARPA-E’s Grid Optimization Competition Challenge 1. The dataset was produced by the same research group responsible for the ACTIVSg family of synthetic grids (Birchfield et al. 2017), using a methodology that statistically matches real US grid properties including degree distribution, average clustering, mean path length, and transmission-line electrical parameters.

The network is represented as an undirected simple graph $G = (V, E)$, where buses are nodes and in-service transmission lines are edges. The MATPOWER case file encodes bus data (bus index, load, voltage level, type) and branch data (terminal buses, impedance, thermal rating, status). Parallel circuits connecting the same bus pair are common in real transmission systems (they provide redundant capacity and fault tolerance); for this structural analysis, parallel branches are reduced to a single edge, retaining the smaller-resistance circuit.

Table 1: Network statistics for the GOC 2,000-bus synthetic US transmission case.

Statistic	Value
Total buses	2,000
Active transmission lines (branches)	3,633
Out-of-service lines (excluded)	6
Parallel circuits collapsed to single edge	827
Unique edges retained	2,806
Connected components	1
Mean degree $\langle k \rangle$	2.806
Maximum degree	11
Graph density	0.00140

The low mean degree (2.81) reflects the practical constraint that each substation connects to only a few neighbors, even in a dense regional network. Transmission voltages in the case span from 13.8 kV to 345 kV, with 69 kV and 138 kV buses most prevalent, reflecting the multi-voltage structure of real grids. This analysis treats all buses as topologically equivalent regardless of voltage level.

4 Methods

4.1 Graph Construction

Bus-branch data is parsed from the MATPOWER format and loaded into an undirected NetworkX graph (Hagberg et al. 2008). Of the 3,639 branches in the case file, 6 are listed as out-of-service and excluded. Parallel circuits (827 pairs) are collapsed to single edges — appropriate for structural analysis, which is insensitive to parallel capacity. The resulting graph has 2,000 nodes and 2,806 edges.

4.2 Degree Distribution

The degree sequence $\{k_i\}$ is characterized by its complementary cumulative distribution function (CCDF): $P(K \geq k)$ as a function of k . The CCDF avoids binning artifacts that distort histograms of heavy-tailed distributions. A power-law degree distribution $p(k) \propto k^{-\gamma}$ implies a CCDF of $P(K \geq k) \propto k^{-(\gamma-1)}$, which is linear in log-log space. A linear fit to $\log k$ versus $\log P(K \geq k)$ yields the slope $\hat{\beta}$, from which the power-law exponent is estimated as $\hat{\gamma} = 1 - \hat{\beta}$. The coefficient of determination R^2 assesses goodness of fit; high R^2 supports a power-law interpretation.

4.3 Small-World Metrics

Two statistics are computed on the largest connected component: the global average clustering coefficient

$$C = \frac{1}{N_{\text{LCC}}} \sum_i C_i,$$

where C_i is the fraction of closed triangles among node i 's neighbors; and the average shortest path length L , the mean geodesic distance over all pairs. These are compared to expected values for an Erdős-Rényi random graph with the same N and mean degree (Watts and Strogatz 1998):

$$C_{\text{rand}} \approx \frac{\langle k \rangle}{N}, \quad L_{\text{rand}} \approx \frac{\ln N}{\ln \langle k \rangle}.$$

The small-world coefficient $\sigma = (C/C_{\text{rand}})/(L/L_{\text{rand}})$ summarizes both deviations; $\sigma \gg 1$ indicates small-world behavior.

4.4 Betweenness Centrality

Node betweenness centrality $g(v)$ is the fraction of all-pairs shortest paths that pass through v , normalized so $g(v) \in [0, 1]$. Nodes with high betweenness are structural bottlenecks whose removal disrupts inter-node routing disproportionately. Betweenness is computed using Brandes' algorithm as implemented in NetworkX.

4.5 Robustness Simulation

Two removal protocols are applied to the LCC:

- **Random failure:** nodes removed in a uniformly random order (seed 42 for reproducibility).
- **Targeted attack:** nodes removed in descending order of initial degree (static — degree ranks are not recomputed after each removal).

After each removal, the size of the remaining largest connected component S is recorded. The robustness curve S/N_0 versus removal fraction f compares the two protocols. Steeper decline under targeted attack relative to random failure confirms the “robust yet fragile” asymmetry predicted for heterogeneous networks (Albert et al. 2000).

5 Results

5.1 Degree Distribution

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(a) Complementary CDF of the degree sequence on a log–log scale. The dashed line is the best-fit power law over the full CCDF. Curvature at high degree indicates a truncated distribution — typical of engineered networks where physical constraints cap the number of connections a substation can terminate.

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(b)

Figure 1

The mean degree is 2.81 and the maximum is 11, indicating modest degree heterogeneity. The degree distribution is right-skewed — most buses have two to four connections, with a thin tail of higher-degree hubs.

The power-law CCDF fit yields $\hat{\gamma} = 4.13$ with $R^2 = 0.79$. For canonical scale-free networks (Barabási–Albert model), $2 < \gamma < 3$ with R^2 near 1. The moderate R^2 here and visible curvature at high degree indicate that the distribution is not purely scale-free. This is consistent with how transmission systems are built: substations are sized by engineering planning and cost optimization, not by the unbounded preferential attachment that generates true power laws. The result is a truncated degree distribution — heavier-tailed than Poisson, but not the pure scale-free form.

5.2 Small-World Properties

Table 2: Small-world metrics for the largest connected component versus an equivalent Erdős–Rényi random graph.

Metric	Observed	Random graph	Ratio
Avg. clustering C	0.0628	0.0014	$44.8\times$
Avg. path length L	16.38	7.37	$2.22\times$
(small-world coeff.)	20.1	—	—

The clustering coefficient of the grid vastly exceeds the random-graph expectation (see Table 2). This reflects the physical structure of regional transmission systems: substations at the same voltage level and geographic area are often meshed together in ring configurations for redundancy, so the neighbors of a bus are likely connected to each other. Despite this local density, the average path length is only modestly longer than the random-graph expectation — the grid’s long-distance high-voltage backbone acts as a hub layer that shortens inter-regional paths. The small-world coefficient $\sigma \gg 1$ confirms the small-world classification.

Watts and Strogatz (1998) found a similar pattern in the 4,941-node Western US power grid ($L = 18.7$, $C = 0.080$, versus $L_{\text{rand}} = 12.4$, $C_{\text{rand}} = 0.005$). The synthetic case studied here reproduces that qualitative structure — high clustering, moderate path lengths — at a different scale and with a different degree sequence.

5.3 Betweenness Centrality

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Figure 2: Top 20 buses by normalized betweenness centrality. Bar color encodes voltage level (kV). High-betweenness buses are structural bottlenecks: they lie on a disproportionate fraction of all shortest paths and are natural candidates for infrastructure hardening.

Betweenness centrality is highly concentrated (Figure 2). The top bus accounts for a substantial fraction of all shortest paths in the network. A handful of high-degree, high-betweenness buses bridge the regional clusters visible in the network visualization. These are the natural targets of any centrality-aware hardening strategy: their removal would force the majority of inter-substation routes to detour through longer, potentially over-capacity alternatives.

5.4 Network Topology

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Figure 3: Spectral-layout visualization of the 2,000-bus LCC. Node color indicates degree (brighter = higher); node size scales with betweenness centrality (larger = more central). Edges are thin gray lines. The spectral embedding reveals coarse community structure: densely connected clusters linked by a backbone of high-betweenness hub nodes.

The spectral layout (Figure 3) reveals a structure of dense regional clusters linked by a sparse backbone. Hub nodes — larger and brighter — sit preferentially at cluster boundaries, consistent with the high betweenness centrality values. The visual impression matches the small-world interpretation: tight local meshes connected by a small number of long-range bridges.

5.5 Robustness Under Failure

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Figure 4: Giant component fraction versus removal fraction for targeted attack (highest-degree first, static) and random failure (uniform random order, seed 42). The shaded region highlights the separation between protocols. The much steeper decline under targeted attack reflects the grid’s structural vulnerability to deliberate disruption of high-degree hubs.

The robustness curves diverge sharply from the first removals (Figure 4). Under targeted attack — removing the highest-degree bus first, then the next, and so on — the giant component fraction drops below 50% after removing only 11.2% of nodes. This rapid fragmentation occurs because the highest-degree buses are bridging hubs: removing them disconnects entire regional clusters from the rest of the network.

Under random failure, the grid retains a large connected component through much of the removal sequence. Randomly selected buses are overwhelmingly likely to be low-degree nodes — degree 2 or 3 — whose removal has local rather than global effect. The giant component declines slowly until the fraction removed is large enough to begin hitting hubs by chance.

This asymmetry — robustness to random failure, fragility to targeted attack — is the signature prediction of the “robust yet fragile” theorem for networks with heterogeneous degree distributions

(Albert et al. 2000). The power grid does not need to be strictly scale-free to exhibit this behavior; degree heterogeneity alone is sufficient.

6 Discussion

The results yield a consistent structural portrait. The degree distribution is heterogeneous — more so than a Poisson (random graph) distribution — but not the clean power law of networks grown by preferential attachment. The fitted exponent $\hat{\gamma} = 4.13$ falls within the range reported in prior power grid studies, but the moderate R^2 and visible high-degree truncation indicate that the distribution is better described as an exponentially truncated power law than a pure scale-free one. This is expected: real substations have hard upper limits on the number of transmission lines they can terminate, set by equipment cost, land area, and engineering standards. No substation approaches the degree that preferential attachment would predict for a scale-free hub.

The small-world result is more clear-cut. The clustering coefficient far exceeds the random-graph expectation while path length remains moderate. The mechanism is straightforward: regional transmission systems are designed with local ring buses and meshed configurations for reliability (boosting clustering), while the long-distance 345 kV and 500 kV backbone corridors act as shortcuts connecting regional clusters (keeping path lengths short). This design is common across US transmission systems and explains why the small-world property appears robustly across different synthetic and real grid topologies.

The robustness asymmetry is arguably the most operationally significant result. Targeted attack causes rapid fragmentation while random failure is well-tolerated. The implication for grid security is direct: a small number of high-degree, high-betweenness substations are responsible for maintaining the network’s connected structure. Physical hardening, cyber security investment, and redundancy planning for this small subset would provide asymmetric resilience benefit relative to uniform infrastructure investment.

Caveats. Three limitations bound these findings. First, the dataset is synthetic: despite statistical matching to real US grids, specific topology details — which buses are most critical, exact path lengths — cannot be mapped to any real substation. The real WECC topology, under CEII protection, may differ structurally in ways the synthetic case does not capture. Second, the robustness simulation models pure topological failure: a node disappears, its edges are removed, and connectivity is recomputed. Real cascades propagate electrically — removing one line overloads adjacent lines, causing protective relay trips that propagate failures faster and farther than topology alone predicts (Buldyrev et al. 2010). Third, betweenness centrality uses shortest geodesic paths, not physical power flows. A flow-betweenness metric based on DC power flow would be more appropriate for operational risk assessment but requires the full admittance matrix and is considerably more expensive to compute.

7 Conclusion

The synthetic 2,000-bus US transmission grid studied here is clearly a small-world network, with clustering far above and path lengths only moderately above their random-graph counterparts. The degree distribution is heterogeneous and broadly consistent with a truncated power law, though not the pure scale-free form. A small fraction of high-degree, high-betweenness buses

bridges the regional clusters that make up the network and is responsible for maintaining global connectivity. Targeted attack on these hubs causes rapid fragmentation; random failure does not.

The structural properties identified here reflect design principles common to real transmission systems: dense local meshing for reliability, sparse long-range interconnections for efficiency. They also reflect a familiar engineering tension: the same topology that makes the grid robust to the random failures that dominate normal operations makes it fragile to the deliberate, intelligence-guided attacks that define advanced threat scenarios. Understanding this tradeoff structurally — before any knowledge of specific substation vulnerabilities — is what network science contributes to infrastructure protection.

8 References

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